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Nonequilibrium Thermofield Dynamics

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The dissipative effects in nonequilibrium thermodfield dynamics are the gauge fields of the $SU(1, 1)$ symmetry of the free bosonic thermal theory [$SU(2)$ for the fermionic one]. In two dimensions some nonequilibrium systems are equivalent to equilibrium systems. An interesting relation exists between the equivalence principle of general relativity and the assumption, in statistical mechanics, of the existence of local subsystems in equilibrium.

1. INTRODUCTION

Thermofield dynamics (TFD) (Takahashi and Umezawa, 1975; Landsman and van Weert, 1987) is a real-time formulation for finite-temperature field theory. One of its advantages is that it makes finite-temperature calculations analogous to zero-temperature ones. It has been applied to field theory and to string and superstring theories (Leblanc, 1987; Ahmed, 1987). Recently TFD has been extended to nonequilibrium systems (Umezawa and Yamanaka, 1988; Matsumoto, 1987). This topic is now called nonequilibrium thermofield dynamics (NETFD). Using NETFD, both the master equation and the dissipation coefficients are derived from the self-consistent renormalization condition.

In this paper some points related to NETFD are discussed. In Section 2 a brief introduction to NETFD is given. In Section 3 it is shown that the dissipative effects can be described as the gauge field of an $SU(1, 1)$ [$SU(2)$] symmetry of the free bosonic (fermionic) thermal theory. In Section 4 the path integral approach to NETFD is used to show that in two dimensions, nonequilibrium effects can be gauged away using reparametrization and conformal invariances. Consequently, an equivalence between nonequilibrium and equilibrium systems is shown. Furthermore, a relation is established between the equivalence principle of general relativity and the

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assumption that statistical systems, even nonequilibrium ones, contain subsystems which are in thermal equilibrium. In Section 5 the conclusions are summarized.

2. A BRIEF REVIEW OF NETFD

In TFD the ordinary vacuum state $|0\rangle$ is replaced by a thermal vacuum $|0(\beta)\rangle$ such that the thermal average of any operator A is

$$\langle A\rangle \neq \langle 0(\beta)|A|0(\beta)\rangle \tag{2.1}$$

This is achieved by doubling the operator space $(a, a^\dagger)$ into $(a, a^\dagger, \tilde{a}, \tilde{a}^\dagger)$ (the quantum numbers are suppressed for the moment), such that

$$\begin{gathered}[a, a^\dagger]_\sigma = 1, \qquad [\tilde{a}, \tilde{a}^\dagger]_\sigma = 1 \\ [a, \tilde{a}]_\sigma = [a, \tilde{a}^\dagger]_{\sigma=0}\end{gathered} \tag{2.2}$$

where

$$[A, B]_\sigma \equiv AB - \sigma BA$$

$\sigma = 1$ (-1) for bosons (fermions). The operators $(a, a^\dagger, \tilde{a}, \tilde{a}^\dagger)$ are not the creation and annihilation operators of $|0(\beta)\rangle$; hence, a Bogoliubov transformation is used to obtain the thermal creation and annihilation operators $(\zeta, \zeta^\dagger, \tilde{\zeta}, \tilde{\zeta}^\dagger)$

$$\zeta|0(\beta)\rangle = \tilde{\zeta}|0(\beta)\rangle = 0 \tag{2.3}$$

Henceforth, the thermal doublet notation ζ^α, $\bar{\zeta}^\alpha$, $\alpha = 1, 2$, will be used, where

$$\zeta^1 = \zeta, \qquad \zeta^2 = \tilde{\zeta}^\dagger, \qquad \bar{\zeta}^1 = \zeta^\dagger, \qquad \bar{\zeta} = -\sigma\tilde{\zeta} \tag{2.4}$$

The evolution of the thermal doublet is shown to be

$$\zeta^\alpha(t) = E^{\alpha\gamma}(t)\zeta^\gamma, \qquad \bar{\zeta}^\alpha(t) = \bar{\zeta}^\gamma(E^{-1}(t))^{\gamma\alpha} \tag{2.5}$$

where

$$E(t) = \exp -i\int_{t_i}^{t} d\rho\,[\omega(\rho) - i\kappa(\rho)\tau_3] \tag{2.6}$$

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

t_i is some initial time, $\omega(t)$ represents the energy, and $\kappa(t)$ is the dissipative coefficient. From (2.5) the evolution of $a^\alpha(t)$, $\bar{a}^\alpha(t)$ is found in the form

$$\begin{gathered}a^\alpha(t) = (B^{-1}(t)E(t))^{\alpha\gamma}\zeta^\gamma \\ \bar{a}^\alpha(t) = \bar{\zeta}^\gamma(E^{-1}(t)B(t))^{\gamma\alpha}\end{gathered} \tag{2.7}$$

where $B(t)$ is the matrix of the Bogoliubov transformation

$$B(t)=\begin{bmatrix}1+\sigma n(t) & -n(t)\\ -\sigma & 1\end{bmatrix} \tag{2.8}$$

and $n(t)$ is the number density

$$n(t)=\langle 0(\beta)|\bar{a}^1(t)a(t)|0(\beta)\rangle \tag{2.9}$$

The semi-free Hamiltonian compatible with the evolution equation (2.7) is $\hat{H}_0(t)$

$$\hat{H}_0=\bar{a}^\alpha(t)[\omega(t)\delta^{\alpha\gamma}-iP^{\alpha\gamma}(t)]a^\gamma(t) \tag{2.10}$$

$$\begin{aligned}P(t)=\kappa(t)&\begin{bmatrix}1+2\sigma n(t) & -2n(t)\\ 2\sigma[1+2\sigma n(t)] & -[1+2\sigma n(t)]\end{bmatrix}\\ &+\sigma\dot{n}(t)\begin{bmatrix}1 & -\sigma\\ \sigma & 1\end{bmatrix}\end{aligned} \tag{2.11}$$

For the case of a field $\psi(x)$ the semi-free Hamiltonian is

$$\hat{H}_0(t)=\int d^3x\,\bar{\psi}^\alpha(x)[\omega(t,-i\nabla)\delta^{\alpha\gamma}-iP^{\alpha\gamma}(t,-i\nabla)]\psi^\gamma(x)$$

where

$$\begin{aligned}\psi^\alpha(x)&=\int d^3k/(2\pi)^{3/2}\,a_k^\alpha(t)\exp i\mathbf{k}\cdot\mathbf{x}\\ \bar{\psi}^\alpha(x)&=\int d^3k/(2\pi)^{3/2}\,\bar{a}_k^\alpha(t)\exp -i\mathbf{k}\cdot\mathbf{x}\end{aligned} \tag{2.13}$$

The master equation and the equations determining $\omega(t)$ and $\kappa(t)$ are determined in NETFD via self-consistent renormalization. The procedure is to calculate on-shell self-energy using fully renormalized fields, and hence set it equal to zero. This procedure has been used successfully to study thermal systems with and without reservoir (Umezawa *et al.*, 1987) to obtain both the master equations and the equations determining ω and κ.

3. DISSIPATIVE EFFECTS AS A GAUGE FIELD

In all the systems considered so far in NETFD the thermal Lagrangian is

$$\hat{\mathscr{L}}=\int d^3k\,\{\bar{a}_k^\alpha(t)[i\partial_t-\omega_k^0]a_k^\alpha(t)-W(\bar{a}^1,a^1)+W(a^2,-\sigma\bar{a}^2)\} \tag{3.1}$$

where W is the interaction term. The unperturbed semi-free Lagrangian is

$$\hat{\mathscr{L}}_0 = \int d^3k \, \{\bar{a}_k^\alpha(t)[i\partial_t - \omega_k + iP_k]^{\alpha\gamma} a_k^\gamma(t)\} \tag{3.2}$$

where ω_k is the renormalized energy and

$$P_k(t) = \begin{bmatrix} 1+2\sigma n_k(t) & -2n_k(t) \\ 2\sigma[1+2\sigma n_k(t)] & -[1+2\sigma n_k(t)] \end{bmatrix} \kappa(t) + \sigma \dot{n}_k(t) \begin{bmatrix} 1 & -\sigma \\ \sigma & 1 \end{bmatrix} \tag{3.3}$$

The covariant derivative $D(t)$

$$D(t) \equiv \partial_t + i[\omega(t) - iP(t)] \tag{3.4}$$

indicates that the term $\omega - iP$ can be considered as a gauge field. To explain this further, consider the Lagrangian

$$\hat{\mathscr{L}}_* \equiv \int d^3k \, \bar{a}_k^\alpha(t) i \, \partial_t a_k^\alpha(t) \tag{3.5}$$

This Lagrangian is invariant under the transformation

$$a^\alpha \to U^{\alpha\gamma} a^\gamma, \qquad \bar{a}^\alpha \to \bar{U}^{\gamma\alpha} \bar{a}^\gamma \tag{3.6}$$

where

$$U = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}, \qquad \bar{U} = \begin{bmatrix} u_{11}^* & -\sigma u_{21}^* \\ -\sigma u_{12}^* & u_{22}^* \end{bmatrix} \tag{3.7}$$

For $\sigma = 1$ the group in (3.6) is $U(1,1)$. For the fermionic case $\sigma = -1$ the group is $U(2)$. The Lie algebra of both cases can be expressed using the four 2×2 matrices $\sigma^\mu \equiv (I, \boldsymbol{\sigma})$, where I is the identity and $\boldsymbol{\sigma}$ are Pauli matrices.

Gauging the symmetry (3.6), i.e., considering U to depend on t, the symmetry can be preserved by replacing the derivative $\boldsymbol{\partial}_t$ by a covariant derivative $D(t)$ defined by

$$D(t) = \partial_t + i\phi(t) \tag{3.8}$$

where $\phi(t)$ is a Lie algebra-valued function in the representation suitable for the operators $a^\alpha(t)$; hence

$$\phi(t) = \phi^\mu(t)\sigma^\mu \tag{3.9}$$

As is well known in gauge theories (Huang, 1983), gauge invariance requires that (3.6) generalizes to

$$\begin{gathered} a \to U(t)a, \qquad \bar{a} \to \bar{a}\bar{U}(t) \\ \phi \to \bar{U}\phi U - i\bar{U}\partial_t U \end{gathered} \tag{3.10}$$

Identifying (3.4) with (3.8), one gets

$$\begin{aligned} \phi^0 &= \omega \\ \phi^1 &= -i\kappa(n+\sigma) \\ \phi^2 &= -(\dot{n}+3\kappa n+\kappa\sigma) \\ \phi^3 &= i\kappa(1+2\sigma n)+i\sigma\dot{n} \end{aligned} \tag{3.11}$$

From (3.11) one concludes that the dissipative effects in NETFD can be expressed as the gauge field of the $SU(1,1)$ or $SU(2)$ gauge invariance of the theory. Notice that only the t direction is gauged; hence, the gauge field is a space-time scalar, and consequently there is no field strength.

4. NONEQUILIBRIUM SYSTEMS IN TWO DIMENSIONS

I will use the path integral formulation of NETFD (Arimitsu *et al.*, 1986; Guida *et al.*, 1987). In Guida *et al.* (1987) a scalar field ϕ model has been studied. The Hamiltonian is given by

$$H(\pi, \phi) = \tfrac{1}{2}\alpha(x)[\pi^2(x)+(\nabla\phi)^2] \tag{4.1}$$

where

$$\alpha(x) = \beta(\mathbf{x})\Theta(t_i - t)+\Theta(t-t_i) \tag{4.2}$$

$\beta(\mathbf{x})$ is the dimensionless, spatially inhomogeneous initial distribution of temperature and Θ is the step function. After integrating out the conjugate momenta $\pi(x)$, one gets the action

$$S = \tfrac{1}{2}\int d^dx\sqrt{-g}\, g^{\alpha\gamma}\,\partial_\alpha\phi(x)\,\nabla_\gamma\phi(x) \tag{4.3}$$

where

$$g^{\alpha\gamma} = \begin{bmatrix} [\alpha(x)]^{-2} & 0 \\ 0 & I_{d-1} \end{bmatrix} \tag{4.4}$$

and I_{d-1} is the identity matrix in $d-1$ dimensions. The formula (4.3) is interesting since it shows a definite resemblance between the effect of an initial inhomogeneous temperature distribution and that of gravity. An analogy between equilibrium temperature effects and gravity has been shown in Laflamme (1988).

Recalling that the equivalence principle in general relativity (Weinberg, 1972) states that locally one can approximate $g_{\alpha\gamma}$ by $\eta_{\alpha\gamma}$ and applying this principle to the system (4.3), I conclude that in nonequilibrium statistical systems there are subsystems which are in thermal equilibrium. This is one of the basic hypothesis of both equilibrium and nonequilibrium statistical mechanics (Zubarev 1984). However, until now it has been only a hypothesis. Now, since the equivalence principle is well founded, the status of the hypothesis of the existence of local subsystems in equilibrium becomes a well-founded principle thanks to NETFD.

In two dimensions the action (4.3) takes the form

$$S=\frac{1}{2}\int d^2x\sqrt{-g}\,g^{\alpha\gamma}\,\partial_\alpha\phi\,\partial_\gamma\phi \tag{4.5}$$

This action has both reparametrization and conformal invariances

$$\begin{aligned}\delta g_{\alpha\gamma}&=\partial_\alpha\zeta_\gamma+\partial_\gamma\zeta_\alpha\\ g_{\alpha\gamma}&\to h^2(x)g_{\alpha\gamma}\end{aligned} \tag{4.6}$$

The three parameters ζ_α, h are sufficient to set $g_{\alpha\gamma}$ in the conformal gauge

$$(g_{\alpha\gamma})=(\eta_{\alpha\gamma})=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \tag{4.7}$$

Therefore, in two dimensions any inhomogeneity in the temperature can be gauged away and the system described by (4.5) is equivalent to the system

$$S=\frac{1}{2}\int d^2x\,\eta^{\alpha\gamma}\,\partial_\alpha\phi\,\partial_\gamma\phi \tag{4.8}$$

which is in thermal equilibrium.

This argument works, provided the theory described by (4.5) is free of conformal anomaly.

The equivalence between nonequilibrium and equilibrium systems does not exist in dimensions other than two, since the system (4.3) does not have conformal invariance except in two dimensions.

This equivalence is also valid for string theory (Schwartz, 1982; Atick and Witten, 1988), which is given by the action

$$S=\frac{1}{2}\int d^2x\sqrt{-g}\,g^{\alpha\gamma}\,\partial_\alpha X^\mu\,\partial_\gamma X_\mu \tag{4.9}$$

where X_μ is a two-dimensional scalar and a D-dimensional vector. In this case the equivalence should be understood only in the two-dimensional sense.

5. CONCLUSION

The dissipative effects have been shown to be the gauge field of the $SU(1,1)$ symmetry of the semi-free bosonic theory, and similarly for the fermionic case, where the gauge symmetry is $SU(2)$.

The existence of local subsystems in thermal equilibrium within non-equilibrium systems is a principle and not just a hypothesis.

In two dimensions the effects of an initially spatially inhomogeneous temperature distribution in a bosonic system can be gauged away provided that the system is free of conformal anomaly. In this case the system is equivalent to another system in thermal equilibrium. An analogous situation occurs for gravity in two dimensions, where the reparametrization and conformal invariances gauge away all the gravitational effects and make any system described by the Hilbert action flat.

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